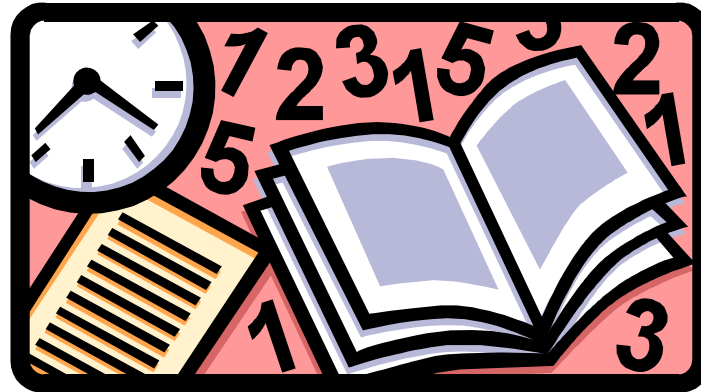




CHURCH WALK CE PRIMARY SCHOOL

Calculations Policy



March 2015

Progression towards a standard written method of calculation



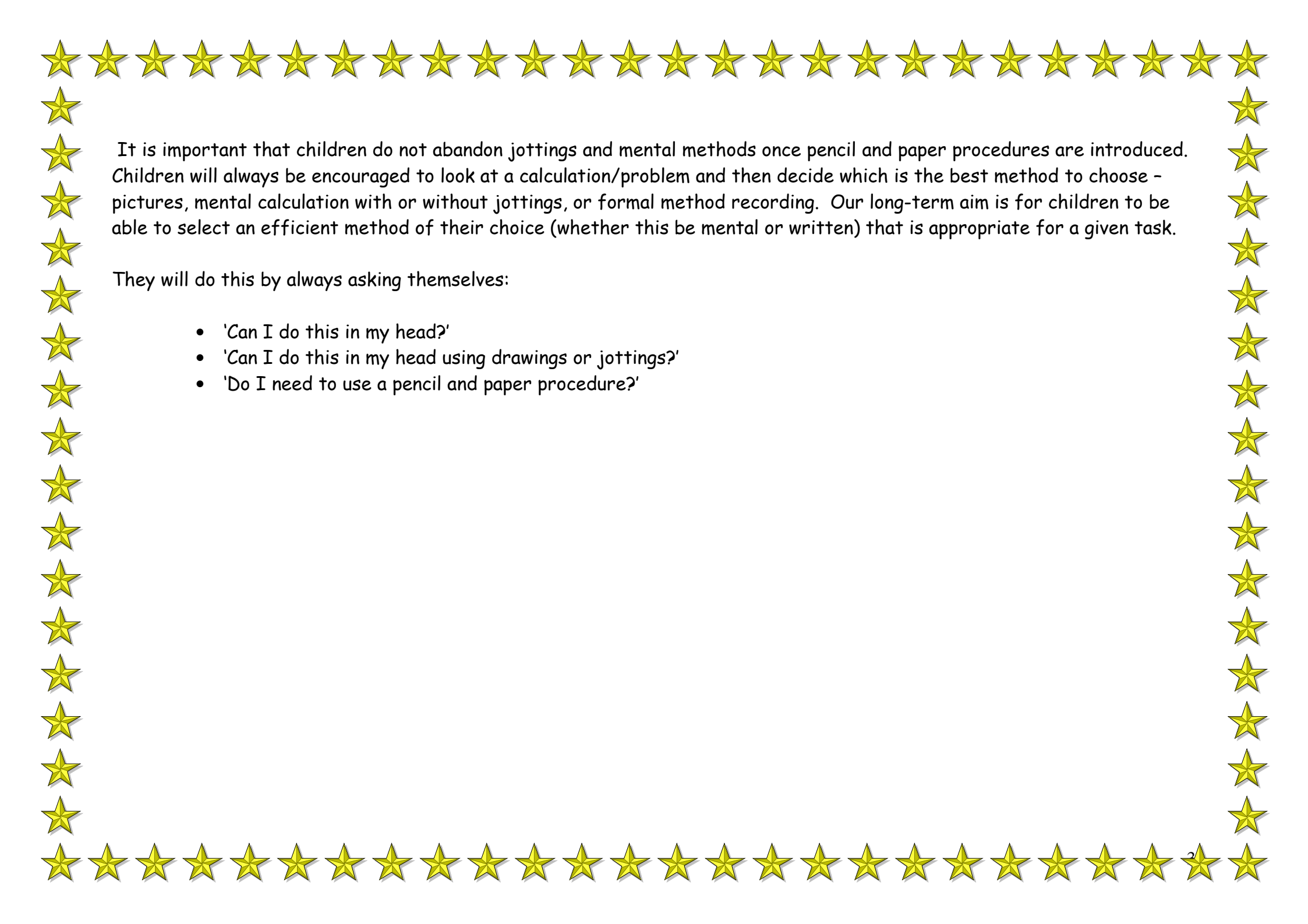
Introduction

At Church Walk, children are introduced to the processes of calculation through practical, oral and mental activities. As children begin to understand the underlying ideas, they develop ways of recording to support their thinking and calculation methods. They use particular methods that apply to special cases, and learn to interpret and use the signs and symbols involved. Over time children learn how to use models and images, such as empty number lines, and use of Dienes apparatus to support their mental and informal written methods of calculation.

There is a considerable emphasis on teaching mental calculation strategies. We do this through 'Big Maths'. Informal written recording takes place regularly and is an important part of learning and understanding. More formal written methods follow only when the child is able to use a wide range of mental calculation strategies. As a child's mental methods are strengthened and refined, so too are their informal written methods. Some recording takes the form of jottings, which are used to support children's thinking. This may be done on scrap paper and whiteboards and is not always retained as it is for the children's own personal use.

This policy contains the key pencil and paper procedures that will be taught within our school. It has been written to ensure consistency and progression throughout the school and reflects a whole school agreement.

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It is important that children do not abandon jottings and mental methods once pencil and paper procedures are introduced. Children will always be encouraged to look at a calculation/problem and then decide which is the best method to choose - pictures, mental calculation with or without jottings, or formal method recording. Our long-term aim is for children to be able to select an efficient method of their choice (whether this be mental or written) that is appropriate for a given task.

They will do this by always asking themselves:

- 'Can I do this in my head?'
- 'Can I do this in my head using drawings or jottings?'
- 'Do I need to use a pencil and paper procedure?'



The overall aim is that when children leave primary school they: -

- ☺ have a secure knowledge of number facts and a good understanding of the four operations;
- ☺ are able to use this knowledge and understanding to carry out calculations mentally;
- ☺ to apply general strategies when using one-digit and two-digit numbers and particular strategies to special cases involving bigger numbers;
- ☺ make use of diagrams and informal notes to help record steps and part answers when more than one calculation is needed to solve a problem;
- ☺ have an efficient, reliable, compact written method of calculation for each operation that children can apply with confidence when undertaking calculations that they cannot carry out mentally

ADDITION

Aims: -

The aim is that children use mental methods when appropriate, but for calculations that they cannot do in their heads they use an efficient written method accurately and with confidence. Children are taught to and should acquire secure mental methods of calculation and one efficient written method of calculation for addition, which they know they can rely on when mental methods are not appropriate.

To add successfully, children need to be able to:

- find 1 more than a given number;
- recall all addition pairs to $9 + 9$ and complements in 10 (e.g. $7 + 3$; $5 + 5$);
- add mentally a series of one-digit numbers, such as $5 + 8 + 4$;
- add multiples of 10 (such as $60 + 70$) or of 100 (such as $600 + 700$) using the related addition fact, $6 + 7$, and their knowledge of place value;
- Partition two-digit and three-digit numbers into multiples of 100, 10 and 1 in different ways.

Key Vocabulary: -

Add, addition, total, plus, more than, and, altogether, increase, equals, make, sum

Consistent use of the vocabulary throughout maths sessions, in conjunction with visual models and practical experiences is essential to developing the concept of addition.

Stage 1 - Through Play

- Using numbers in familiar contexts like number rhymes or in role-play.
- Counting of everyday objects.
- Saying the number names in order.
- Recognise the numerals from 1-9.
- Recognise, count and order numbers up to 20.
- Putting small groups of objects together and counting to find the total.
- Solving simple problems involving the use of the skills listed above.

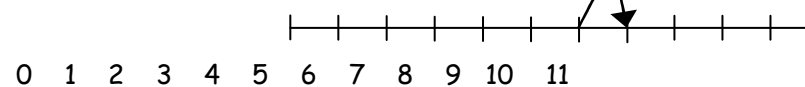
Stage 2 - Marked Number Line

- From the first stages children will introduced to number lines and encouraged to visualise calculation.

7 and 1 **more** is 8

7 **add** 1 equals 8

$7 + 1 = 8$



The children will learn to relate addition to the combining of 2 groups. E.g. Count out 2 cakes. Count out 2 more cakes. How many cakes altogether?

- Children are also encouraged to use their fingers to keep track at this stage.
- Children then begin to use numbered lines to support their own calculations using a numbered line to count on in ones.

$8 + 5 = 13$



ADDITION

Stage 3 - The Empty Number Line

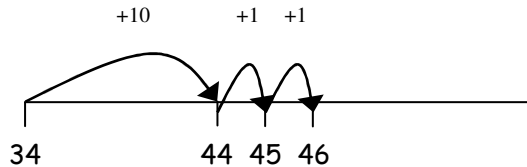
Children will begin to use 'empty number lines' themselves starting with the larger number and counting on.

The empty number line helps to record the steps on the way to calculating the total. This can be used alongside dienes equipment.

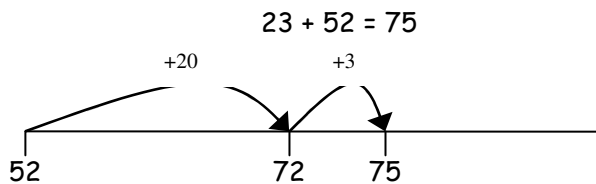
$$34 + 12 = 46$$

Children are encouraged to

- start on the largest number (34).
- partition the second number (10 and 2).
- Count on in tens.
- Count on in 1's.



As confidence increases, children are taught to count larger jumps in their heads and to record the steps they take.



When children are presented with problems they are encouraged to identify the calculation first either through discussion work or by writing down the number sentence.

Older children are encouraged to use this method when calculating in their heads (e.g. $2.3 + 5.2$; $123 + 46$ etc)

Stage 4 - Expanded Column Addition Using Partitioning

The expanded method leads children to the more compact method and is the next step to developing an efficient pencil and paper procedure. The amount of time that should be spent teaching and practising the expanded method will depend on how secure the children are in their recall of number facts and in their understanding of place value.

Horizontal partitioning:

$$\begin{aligned} 74 + 98 &= ? \\ 70 + 90 &= 160 \\ 4 + 8 &= 12 \\ 160 + 12 &= 172 \end{aligned}$$

Children can use this with larger numbers, but this time line up the 10s and 1s. Add the 1s, then the 10s, then the 100s. Recombine the numbers in the answer section.

$$126 + 451 = 577$$

$$\begin{array}{r} 100 \quad 20 \quad 6 \\ + 400 \quad 50 \quad 1 \\ \hline 500 \quad 70 \quad 7 \end{array} \quad 577 \rightarrow$$

and smaller numbers.

$$4.3 + 2.9 = 7.2$$

$$\begin{array}{r} 4 \quad 0.3 \\ + 2 \quad 0.9 \\ \hline 6 \quad 1.2 \end{array} \quad 7.2 \rightarrow$$

ADDITION

Stage 5 - Column Method - Carrying

Children should only go onto this next stage if they are ready and confident:

$$\begin{array}{r} 47 \\ + 76 \\ \hline 123 \\ 11 \end{array} \quad \begin{array}{r} 258 \\ + 87 \\ \hline 345 \\ 11 \end{array} \quad \begin{array}{r} 366 \\ + 458 \\ \hline 824 \\ 11 \end{array}$$

Column addition remains efficient when used with larger whole numbers and decimals of the same length. Once learned and children have become confident with this method it is quick and reliable.

Throughout their experiences in calculation, children should experience units of measure to support problem solving (e.g. £.p; cm, mm, m, km; g, kg; ml, l).

Stage 6 - Column Method - Carrying with Decimals of Different Length

Children should have this modelled to them alongside stage 5, when the need arises.

Children need to remember to line up the decimal point and place the numbers on accordingly.

$$4.6\text{kg} + 2.39\text{kg} = 6.99\text{kg}$$

$$\begin{array}{r} 4.6 \\ + 2.39 \\ \hline 6.99 \end{array}$$

Children may chose to add an extra 0 to help avoid mistakes.

$$\begin{array}{r} 4.60 \\ + 2.39 \\ \hline 6.99 \end{array}$$

SUBTRACTION

Aims: -

The aim is that children use mental methods when appropriate, but for calculations that they cannot do in their heads they use an efficient written method accurately and with confidence. Children are taught to and should acquire secure mental methods of calculation and one efficient written method of calculation for subtraction, which they know they can rely on when mental methods are not appropriate.

To subtract successfully, children need to be able to:

- Count back insteps of 1 and 10 from any number up to 100.
- recall all addition and subtraction facts to 20, 100, 1000;
- subtract multiples of 10 (such as $160 - 70$) using the related subtraction fact, $16 - 7$, and their knowledge of place value.
- partition two-digit and three-digit numbers into multiples of one hundred, ten and one in different ways (e.g. partition 74 into $70 + 4$ or $60 + 14$).

Key Vocabulary: -

Subtract, subtraction, take away, minus, less than, difference, decrease, leave, how many left?

Consistent use of the vocabulary throughout maths sessions, in conjunction with visual models and practical experiences is essential to developing the concept of addition.

Stage 1 - Through Play

- Counting backwards out loud.
- Singing nursery rhymes.
- Counting in real situations.
- Using fingers.
- Taking 1 or more objects away from a group and counting those left.
- Drawing pictures.



- Solving simple problems involving the use of the skills listed above.

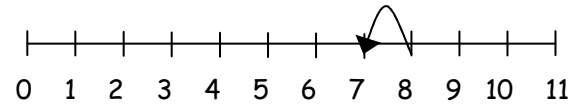
Stage 2 - Marked Number Line

- From the first stages children will introduced to number lines and encouraged to visualise calculation.

1 less than 8 is 7

8 subtract 1 equals 7

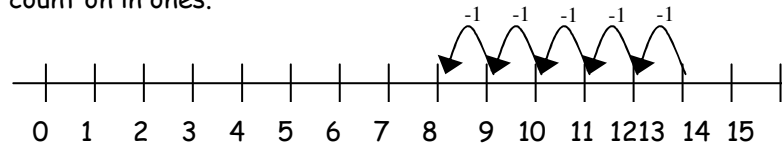
$8 - 1 = 7$



The children will learn to relate subtraction to removing an object from a group e.g. Count out 5 blocks. Take away 2 cakes. How many cakes are left?

- Children are also encouraged to use their fingers to keep track at this stage.
- Children then begin to use numbered lines to support their own calculations using a numbered line to count on in ones.

$13 - 5 = 8$



SUBTRACTION

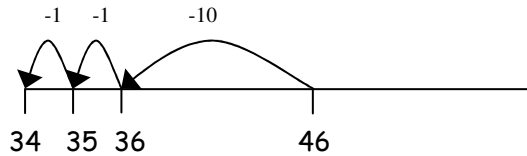
Stage 3 - The Empty Number Line

Children will begin to use 'empty number lines' themselves starting with the larger number and counting on.
The empty number line helps to record the steps on the way to calculating the total.

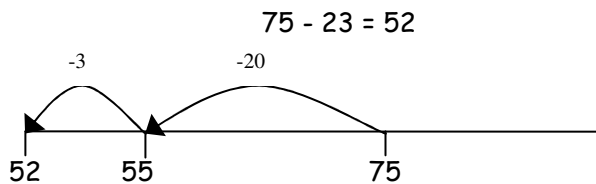
$$46 - 12 = 34$$

Children are encouraged to

- start on the first number in the number sentence (46).
- partition the second number (10 and 2).
- Count back in tens.
- Count back in 1's.



As confidence increases, children are taught to count larger jumps in their heads and to record the steps they take.



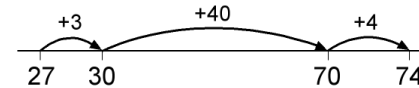
When children are presented with problems they are encouraged to identify the calculation first either through discussion work or by writing down the number sentence.
Older children are encouraged to use this method when calculating in their heads (e.g. $5.2 - 3.2$; $123 - 46$ etc)

Stage 4 - Finding the Difference

Finding the difference should be done by counting up from the smaller to the larger number can be recorded using a number line.

Find the difference between 74 and 27 or $30 + \square = 74$

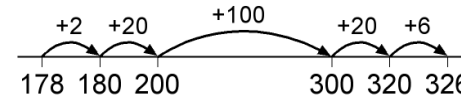
- Start from the smallest number and count to the largest
- Count to the next 10 in 1s (30) and write how many above the jump
- Find the 10s number before the target number (70) and write how many above the jump
- Count on in 1s to the target (74) and write how many above the jump



- To find the answer, children need to add the numbers above the jumps ($3 + 40 + 4$). Answer 47

With three-digit numbers the number of steps can be reduced as children become more proficient with using a number line to track their counting on.

Find the difference between 326 and 178 or $178 + \square = 326$



- Children can reorganise the jumps above the line to add them up more efficiently by putting the steps in order
 $100 + 20 + 20 + 6 + 2 = 148$
- This method is particularly useful when calculating with decimals.

SUBTRACTION

Stage 5 - Expanded Column Subtraction with No Decomposition

The expanded method leads children to the more compact method and is the next step to developing an efficient pencil and paper procedure.

- Horizontally

$$\begin{aligned} 86 - 42 &= ? \\ 86 - 40 &= 46 \\ 46 - 2 &= 44 \end{aligned}$$

- Vertically
- Partition each number and set out taking care to line up 10s and 1s.
- Subtract the 1s, then the 10s.
- Recombine the numbers in the answer section.

$$\begin{array}{r} 86 - 42 = \\ \begin{array}{r} 80 \quad 6 \\ - 40 \quad 2 \\ \hline \end{array} \\ \begin{array}{r} 40 \quad 4 \\ \hline \end{array} \quad 44 \end{array} \rightarrow$$

Children can use this with larger numbers

$$\begin{aligned} 355 - 122 &= 233 \\ \begin{array}{r} 300 \quad 50 \quad 5 \\ - 100 \quad 20 \quad 2 \\ \hline 200 \quad 30 \quad 3 \end{array} &\rightarrow 233 \end{aligned}$$

and smaller numbers.

$$\begin{aligned} 4.9 - 2.3 &= 2.6 \\ \begin{array}{r} 4 \quad 0.9 \\ + 2 \quad 0.3 \\ \hline 2 \quad 0.6 \end{array} &\rightarrow 2.6 \end{aligned}$$

Stage 6 - Expanded Column Subtraction with Decomposition

In some calculations we need to exchange 10s into 1s; 100s into 10s etc. This is called *exchanging*.

$$82 - 36 = 46$$

- Partition each number and set out taking care to line up 10s and 1s.
- Subtract the 1s, then the 10s.
- 2 - 6 cannot be done, so we must exchange a 10 and put it with the ones.
- We must cross out and record how we have adjusted the calculation
- This leaves 12 - 6; and 70 - 30
- Recombine the numbers in the answer section.

$$\begin{array}{r} 82 - 36 = \\ \begin{array}{r} 70 \quad 12 \\ \del{80} \quad \del{2} \\ - 30 \quad 6 \\ \hline \end{array} \\ \begin{array}{r} 40 \quad 6 \\ \hline \end{array} \quad 46 \end{array} \rightarrow$$

Children can use this with larger numbers

$$\begin{aligned} 335 - 192 &= 143 \\ \begin{array}{r} 200 \quad 130 \\ \del{300} \quad \del{30} \quad 5 \\ - 100 \quad 90 \quad 2 \\ \hline 100 \quad 40 \quad 3 \end{array} &\rightarrow 143 \end{aligned}$$

SUBTRACTION

Stage 7 - Column Subtraction with Decomposition

Children should only go onto this next stage if:

- they are ready.
- they are confident (i.e. secure level 4).
- they have completed 'Year 6 SATs' and are being taught this method to support the transfer to secondary school.

$$354 - 183 = 171$$

$$\begin{array}{r} 2 \ 15 \\ \cancel{3}54 \\ - 183 \\ \hline 171 \end{array}$$

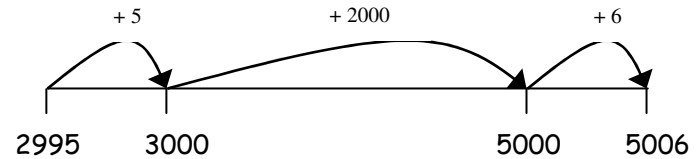
$$6467 - 2684 = 3783$$

$$\begin{array}{r} 5 \ 13 \ 16 \\ \cancel{6}467 \\ - 2684 \\ \hline 3783 \end{array}$$

Stage 8 - Developing Efficient use of the Number Line

In some calculations we need to choose the most efficient method. Sometimes this is the number line.

$$5006 - 2995 = 2011$$



$$2000 + 6 + 5 = 2011$$

MULTIPLICATION

Aims: -

The aim is that children use mental methods when appropriate, but for calculations that they cannot do in their heads they use an efficient written method accurately and with confidence. Children are taught to and should acquire secure mental methods of calculation and one efficient written method of calculation for multiplication, which they know they can rely on when mental methods are not appropriate.

To multiply successfully, children need to be able to:

- Count on in regular steps e.g. 2s, 3s, 5s
- recall all multiplication facts to 10×10 ;
- partition number into multiples of HTU.
- work out products e.g. 70×5 , 70×50 , 700×5 related fact 7×5 and their knowledge of place value;
- add two or more single-digit numbers mentally;
- add multiples of 10 (such as $60 + 70$) or of 100 (such as $600 + 700$) using the related addition fact, $6 + 7$, and their knowledge of place value;
- add combinations of whole numbers using the column method (see above).

Key Vocabulary: -

Multiply, times, product, groups of lots of, multiplied by, x times bigger

Consistent use of the vocabulary throughout maths sessions, in conjunction with visual models and practical experiences is essential to developing the concept of addition.

Stage 1 - Through Play

- count repeated groups of the same size using toys and practical resources e.g. If each child has 2 toys, how many altogether?
- participate in practical sessions, e.g. counting pairs of socks and fingers on hands.
- Introduce counting in 10s.
- Use drama to act out stories involving multiplication.

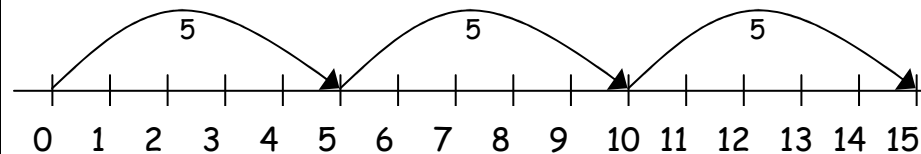
Stage 2 - Repeated Addition

Children to learn that adding groups of the same size is the same as multiplying. This can be recorded with practical resources and pictures, and on a number line. Children should regularly count in steps of the same size e.g. 2's, 5's, 10s and learn doubles up to 10.

3 times 5 is $5 + 5 + 5 = 15$ or 3 lots of 5 or 5×3

Repeated addition can be shown easily on a number line:

$$5 \times 3 = 5 + 5 + 5$$



MULTIPLICATION

Stage 3 - Doubling Numbers Larger than 10

Children need to apply partitioning skills to help them to double numbers. The following model can help to keep track when the numbers are bigger than 10.

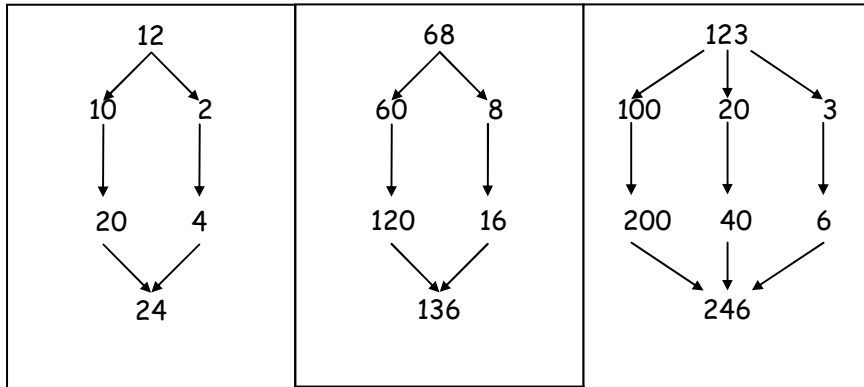
Double 12

- Partition into 10 and 2
- Double each part
- Recombine

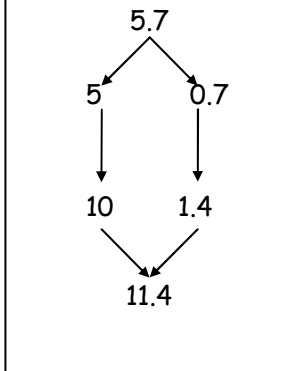
Step 1

leads to

leads to



Can also be used for decimals.

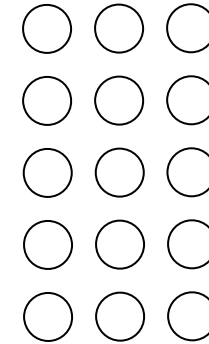
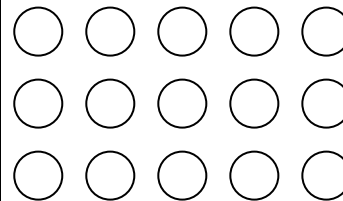


Stage 4 - Multiplication Using Arrays

Arrays can be used to show that multiplication can be done in any order. Children will start by using practical materials (cake tins, multi-link cubes) to develop the concept.

This demonstrates the concept of adding groups of the same size.

$$5 \times 3 = 3 \times 5 = 15$$

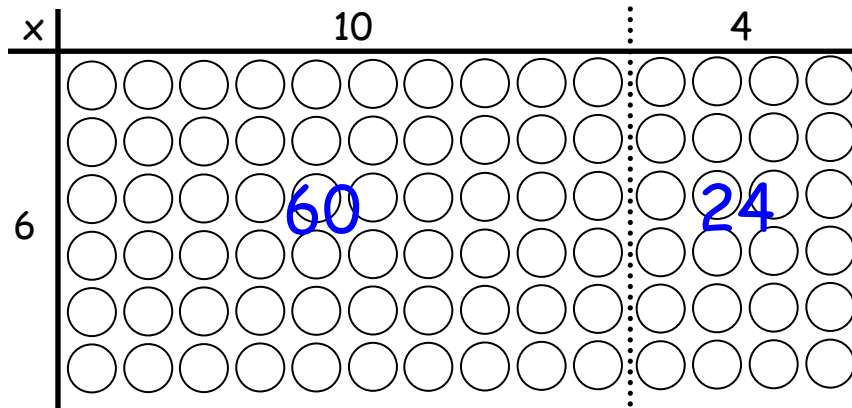


Arrays provide the opportunity to introduce inverse of division-
 $15 \div 3 = 5$ and $15 \div 5 = 3$.

MULTIPLICATION

Stage 5 - From Arrays to Grid Method

By this point children should have a good grasp of the concept of multiplication and should know many multiplication facts. Children should be able to model a multiplication calculation using an array. This knowledge will support with the development of the grid method.



$$6 \times 10 = 60$$

$$6 \times 4 = 24$$

$$60 + 24 = 84$$

The grid method is the natural progression from showing multiplications as arrays. It is an alternative way of recording the same steps.

Stage 6 - Grid Method

The grid method is a way of using partitioning to break multiplication into manageable chunks.

$$14 \times 6 = 84$$

- Set out using a grid
- Partition the numbers (10 and 4)
- Multiply each section of the grid (10 x 6; 4 x 6)
- Add the products to find the answer (60 + 24)

x	10	4	$\begin{array}{r} 60 \\ +24 \\ \hline 84 \end{array}$
6	60	24	

The grid can be developed to multiply TU x TU and HTU x TU.

$$25 \times 17 = 425$$

x	20	5	$\begin{array}{r} 200 \\ 140 \\ 50 \\ +35 \\ \hline 425 \\ 1 \end{array}$
10	200	50	
7	140	35	

MULTIPLICATION

Stage 6 continued -Grid Method with Decimals

The grid can be used with decimals and is useful when calculating with units of measurement.

$$14 \times 0.6\text{kg} = 8.4\text{kg}$$

X	10	4
0.6kg	6	2.4

6.0
<u>+2.4</u>
<u>8.4</u>

$$2.7\text{m} \times 16 = 43.2\text{m}$$

X	2m	0.7m
10	20	7
6	12	4.2

20.0
12.0
7.0
<u>+ 4.2</u>
<u>43.2</u>
1

Stage 7 - Moving to Column Multiplication

This method reduces the recording, showing links to the grid method.

$$56 \times 27 \text{ is approximately } 60 \times 30 = 1800.$$

56	
<u>x 27</u>	
42	(6 x 7)
350	(50 x 7)
120	(6 x 20)
<u>1000</u>	(50 x 20)
<u>1512</u>	
1	

Division

Aims: -

The aim is that children use mental methods when appropriate, but for calculations that they cannot do in their heads they use an efficient written method accurately and with confidence. Children are taught to and should acquire secure mental methods of calculation and one efficient written method of calculation for division, which they know they can rely on when mental methods are not appropriate.

To divide successfully in their heads, children need to be able to:

- Count object into equal groups and in regular steps;
- understand and use the vocabulary of division - for example in $18 \div 3 = 6$, the 18 is the dividend, the 3 is the divisor and the 6 is the quotient;
- partition two-digit and three-digit numbers into multiples of 100, 10 and 1 in different ways;
- recall multiplication and division facts to 10×10 , recognise multiples of one-digit numbers and divide multiples of 10 or 100 by a single-digit number using their knowledge of division facts and place value;
- find a remainder working mentally - for example, find the remainder when 48 is divided by 5;
- understand and use multiplication and division as inverse operations.

Key Vocabulary: -

Divide, division, divided by, share, sharing, equal groups of, equally, how many, remainder, quotient.

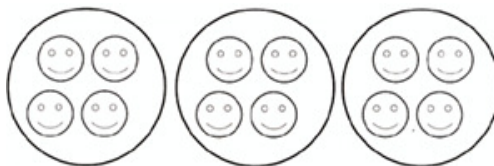
Consistent use of the vocabulary throughout maths sessions, in conjunction with visual models and practical experiences is essential to developing the concept of addition.

Stage 1 - Through Play

- Share toys and practical resources into groups of the same size e.g. If you share 10 biscuits between 2 children, how many will each child have?
- Participate in practical sessions, e.g. counting a collection of objects into groups of equal size.
- Introduce counting in 10s.
- Act out stories involving division.

Stage 2 - Drawing Pictures to Show Sharing

Children will understand equal groups and share items out in play and problem solving. They will count in 2s and 10s and later in 5s.



12 shared between ; 12 shared into 3 equal groups

Children will develop their understanding of division and use jottings to support calculation.

DIVISION

Stage 3 - Grouping or Repeated Subtraction

Children solve division problems by removing equal groups of objects from a collection.

$12 \div 3 = 4$ How many groups of 3 can be made from 12?



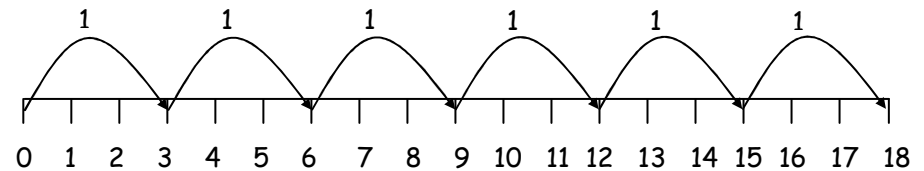
Stage 4 - Division on a Number Line

Children need frequent practice in counting in regular steps. This will help them to use number lines to record division.

Division can be solved by counting on.

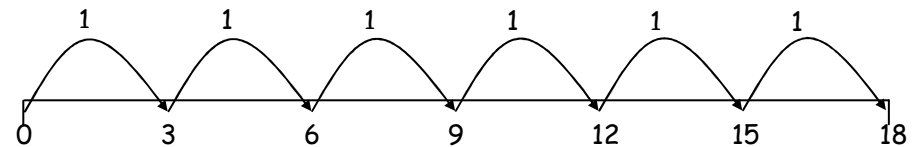
Children need to be taught ways to read division number sentences.

$18 \div 3 =$; How many 3's make 18; How many groups of 3 can be made from 18?



Children find the answer by counting the number of jumps.

As confidence grows, children can use an empty number line to track the number of groups or jumps.



DIVISION

Stage 5 - Halving

Children need to apply partitioning skills to help them to halve numbers. The following model can help to keep track when the numbers are bigger than 10. Initially this model needs to be used with even numbers.

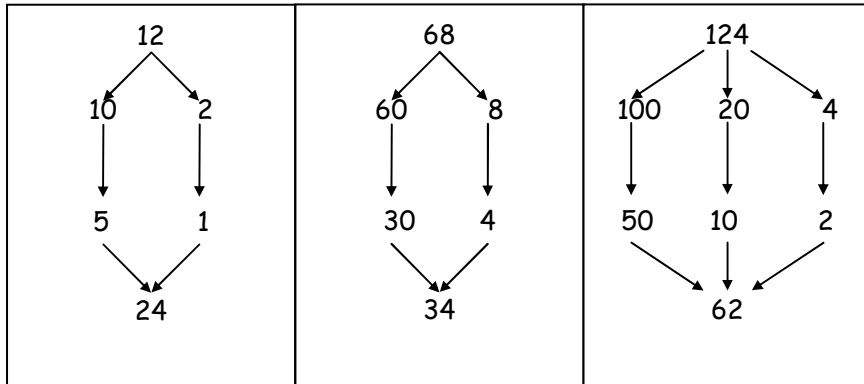
Half 12

- Partition into 10 and 2
- Half each part
- Recombine

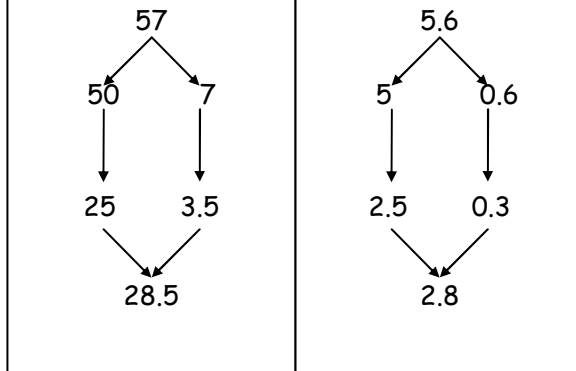
Step 1

leads to

leads to



Can also be used for odd numbers and decimals.



Stage 6 - Remainders

When children know more multiplication facts they will begin to recognise when there will be remainders. The ideas below may be taught alongside visual representations such as arrays or a number line, but most children at this stage should be using knowledge of multiplication facts.

- $27 \div 5 = ?$
- 27 does not end in 5 or 0 therefore there must be a remainder.
- Which is the largest multiple of 5 without going past 27?
- 25 is, so do $25 \div 5 = 5$.
- Now work out the remainder. What is the difference between 25 and 27? 2, so remainder 2
- $27 \div 5 = 5 \text{ r}2$

DIVISION

Stage 7 - Division with larger numbers

Children need to use known facts to support their calculation.

$96 \div 6 = ?$; How many 6's make 96; How many groups of 6 can be made from 96?

$1 \times 6 = 6$ $2 \times 6 = 12$ $10 \times 6 = 60$ $5 \times 6 = 30$
--

Start with a known facts box.

This helps children who have yet to learn all of their tables to access problems involving division.

Now partition the number:

$96 \div 6 = ?$

$96 = 60 + 36$. We have chosen 60 because we are dividing by 6, and it is 10 lots.

$60 \div 6 = 10$

$36 \div 6 = 6$

$10 + 6 = 16$

Do this with larger numbers and remainders:

$127 \div 5 = ?$

$127 = 50 + 50 + 25 + 2$

$50 \div 5 = 10$

$50 \div 5 = 10$

$25 \div 5 = 5$

2 = a remainder

$10 + 10 + 5 + r2 = 25 \text{ r}2$

Stage 8- Chunking

This method relies on children having a solid knowledge of times tables and related division facts

$196 \div 6 = 16$

$$\begin{array}{r}
 192 \\
 -180 \quad (30 \times 6) \\
 \hline
 -12 \quad (2 \times 6) \\
 \hline
 0 \\
 30 + 2 = 32
 \end{array}$$

Extend to larger numbers:

$1466 \div 4 =$

$$\begin{array}{r}
 1466 \\
 -1400 \quad (350 \times 4) \\
 \hline
 -66 \quad (44 \times 4) \\
 \hline
 350 + 44 = 394
 \end{array}$$

$350 + 44 = 394$

Extend to remainders:

$348 \div 7 =$

$$\begin{array}{r}
 348 \\
 -280 \quad (40 \times 7) \\
 \hline
 68 \\
 -63 \quad (9 \times 7) \\
 \hline
 5
 \end{array}$$

$40 + 9 = 49 \text{ r}5$

DIVISION

Stage 9 - The Bus Stop

This method relies on children having a solid knowledge of times tables and should be introduced when they have secured chunking confidently.

$$96 \div 6 = 16$$

$$\begin{array}{r} 16 \\ 6 \overline{)96} \end{array}$$

Extending to remainders: -

$$97 \div 6 = 16 \text{ r } 1$$

$$\begin{array}{r} 16 \text{ r } 1 \\ 6 \overline{)97} \end{array}$$

Remainders as decimals: -

$$87 \div 5 = 17.4$$

$$\begin{array}{r} 17.4 \\ 5 \overline{)87.20} \end{array}$$